

Erratum

Statistical Structures Underlying Quantum Mechanics and Social Science

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Common observations of the unpredictability of human behavior and the influence of one question on the answer to another suggest social science experiments are probabilistic and may be mutually incompatible with one another, characteristics attributed to quantum mechanics (as distinguished from classical mechanics). This paper examines this superficial similarity in depth using the Foulis-Randall Operational Statistics language. In contradistinction to physics, social science deals with complex, open systems for which the set of possible experiments is unknowable and outcome interference is a graded phenomenon resulting from the ways the human brain processes information. It is concluded that social science is, in some ways, “less classical” than quantum mechanics, but that generalized “quantum” structures may provide appropriate descriptions of social science experiments. Specific challenges to extending “quantum” structures to social science are identified.

KEY WORDS: Foulis-Randall Operational Statistics; Fuzzy Trace theory; memory; generalized statistical framework; quantum structures; social science; cognitive framing; packing; subjective probability; quantum mechanics; quantum logic.

1. INTRODUCTION

Human beings are much bigger than electrons, and much more complex. Humans do not change their states by small, discrete amounts. And even though human beings themselves come in discrete packages, their behavior typically does not, at least at the level of analysis used in social science. All of this suggests that descriptions of elementary particles and descriptions of elementary school children are likely to be quite different. I am not therefore suggesting that the same phenomena occur in both, and yet, the roll of a die and proteins crossing a cell membrane are also quite dissimilar, but the language of ordinary statistics can successfully describe certain aspects of both. In this article I examine ways in which social

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science may require generalized statistical understandings akin to understandings necessitated in physics by the advent of quantum mechanics, as well as ways such social science generalizations may be different from those in quantum mechanics.

I offer the perspectives presented here as a practicing social scientist: I have doctorates in both medicine and psychology; I currently work in a department of psychiatry and have spent a number of years in departments of psychology, sociology and communication. I proffer these thoughts in the context of my previous work in quantum logic and Operational Statistics (Wright, 1977a,b; Wright, 1978a,b; Swift and Wright, 1980; Wright, 1990). One important goal is to suggest possible directions for development, if social sciences are to be encompassed by “quantum” structures.

In order to highlight the parallels and contrasts between quantum mechanics and social science, I take specific—and simplified—versions of each. As representative of quantum mechanics I take experiments on spin-one particles, wherein states are unit vectors in the complex Hilbert space \mathbb{C}^3 and observables are Hermitian operators on that Hilbert space. Obviously, this does not characterize all of “quantum logic” or “quantum structures” and, indeed, I argue that “quantum” structures go beyond physics and provide, or at least suggest, the requisites for understanding social science. This is particularly true of the Foulis-Randall Operational Statistics formulation, which I use in much of what follows.

For present purposes, I take social science to mean the study of human behavior. As an illustrative example of social science, I use studies of individual human beings in experimental situations as might occur in cognitive psychology, with basic experiments on memory providing specifics. I have tried to describe below enough of the theory and operationalization of the memory experiments to make the comparison clear, but I have simplified both the theory and the data somewhat in this presentation.

The outline of the paper is as follows: First I sketch reasons one might expect similarities or differences between quantum mechanics and social sciences. Then, to facilitate comparison with social science, I describe the Hilbert space manual of experiments on spin-one particles in the language of Foulis-Randall Operational Statistics. I then review the theory and operationalization of memory experiments in which subjects learn lists of words from common categories (e.g., fruits or animals), describing these also in the Foulis-Randall language. Based on these expositions, I suggest ways in which social science is similar to and different from quantum mechanics at the level of mathematical and statistical description. Finally, I outline what would be needed to extend “quantum” structures to social science.

1.1. Superficial Similarities and Differences

A facile—and I think specious—argument for differences between physics and social science is that the former is an exact science while the latter is inexact.

Physics is exact in that it can predict the outcome of certain experiments with great accuracy (but compare Hedges, 1987). Consider, however, what happens if I drop a feather. While the influences on the feather's descent—gravity, rigidity and mass of the feather, air flow, air temperature, initial linear and angular momentum, distance to the floor, etc.—are all physical concepts, no one can, in reality, predict exactly where the feather will fall. Some might object that we could in principle do this if we made all of the appropriate measures, but we cannot make all these measurements for real feathers in real rooms. Yet we frequently hold social science to the task of predicting how a particular human being—a vastly more complex system than a feather in a room—will behave in a certain situation. This fundamental difference—the complexity of the phenomena to be described and predicted—between physical and social science is an underpinning of much of what follows here. The task of learning words on a list is a very simple task, yet it confronts us with many complications. Certain very low-level tasks such as reaction times in attentional tasks might be simpler (but maybe not) and most tasks of psychology are vastly more complex than memory for word lists.

Nevertheless, there are also many reasons to believe that social science might behave in a “non-classical” way. A key aspect of quantum mechanics is incompatible measurements. In social science, asking one question of a human being can change the state of the organism so that the answers to other questions are altered. This could happen for external reasons (e.g., so as to not appear inconsistent to the questioner) or for internal reasons (e.g., one question might make certain things more, or less, accessible in memory).

Incompatible measurements occur with spin-one particles when spin components are measured along axes, say, 45 degrees apart, if the first measurement (Pauli, 1958, p. 72: “measurements of the first kind”) puts the system in an eigenstate of the measurement operator. It may not be entirely dissimilar if a person facing an emotional situation is first asked “Do you feel angry?” Answering “Yes” may take the person from a blended emotional state to an experience of anger, leading to both another “Yes” answer if asked again (say, by someone else) and to changes in the responses to questions about fear or sadness, a “collapsing of the emotion wave,” if you will.

And both situations may also be inherently probabilistic: Before the first question, it may be neither true nor false that the person is angry; they may be feeling a jumble of things that can get conceptualized as anger or as something else. This would seem, at least in some cases, to go beyond a “mixed state” consisting of 40% anger, plus 35% fear, plus. . . . There could also be an incompatibility in emotional states, so that the more certain one is of being angry, the less certain one is of being afraid, resulting in an “uncertainty principle for emotions.”

With this sense that there are certain “intriguing parallels” between quantum mechanical phenomena and social science phenomena, and yet important differences, let us begin an in-depth comparison of specific examples, starting with a

common language. An ultimate question is whether Nature takes advantage of the incompatibilities in social science operations and produces results, as quantum mechanics does, that are inconsistent with ordinary statistics.

1.2. The Foulis-Randall Descriptive Language

Foulis and Randall's Operational Statistics (e.g., Foulis and Randall, 1972; Randall and Foulis, 1973, 1978) is a generalization of the Kolmogorov (1933, 1956) description of an experiment consisting of a set X of possible outcomes and a σ -field of subsets of X identified as observable events; probability measures on this σ -field are states of the system.

In the Foulis-Randall generalization, a set of experiments, or "operations," is considered together and thought of as the manual or handbook containing descriptions of all of the operations that can be performed on the system under consideration. Each operation is represented by its set of outcomes, so a manual is a set of sets. These sets overlap if one outcome can occur in two distinct experiments. Any subset of an operation is considered an observable event. Events are orthogonal if they operationally reject one another, that is, if they are disjoint and their union is an event. A probability measure, or weight function, ω for a manual is a mapping of outcomes into $[0, 1]$ with an unordered sum of 1 over any operation; the set of all such weight functions is Ω , the weight space for the manual. An operational proposition for a manual is defined via the set of outcomes that confirm the proposition and the set of outcomes that refute the proposition. Taken together, these ordered pairs of outcomes form the logic Π for the manual. Under favorable conditions Π is an orthomodular poset. The probability measures on the logic are called states.

2. A QUANTUM MECHANICS EXAMPLE: SPIN-ONE EXPERIMENTS

Measurements on a spin-one particle correspond to Hermitian operators on \mathbb{C}^3 and can be carried out using a generalized Stern-Gerlach procedure (Swift and Wright, 1980). The outcomes of the corresponding operation are the eigenspaces of the Hermitian operator, which are mutually orthogonal and span the space. Hence, the manual of operations can be taken to be all maximal orthogonal sets of subspaces of \mathbb{C}^3 , or equivalently, all maximal orthogonal sets of projections on \mathbb{C}^3 (cf. Foulis *et al.*, 1983, Examples 12, 13). Real physical measurements are presumed to correspond approximately to these theoretical measurements. Events are orthogonal sets of projections and two events are orthogonal as events exactly when they are orthogonal as sets of projections. A weight function on the manual is a measure on the projection lattice and hence by Gleason's Theorem (Gleason, 1957) corresponds to a density operator on \mathbb{C}^3 .

The logic of the spin-one manual is isomorphic to the projection lattice on \mathbb{C}^3 : Outcomes that confirm the proposition corresponding to projection P are projections less than or equal to P ; outcomes that refute the proposition are projections orthogonal to P . States on the logic are equivalent to weight functions on the manual and therefore to density operators. Note that the physical states go beyond the states on the logic in that they contain phase information. States differing only in their phase information correspond to the same weight function on the manual, though they differ in the effect they have in compound operations (Wright, 1978a).

One potentially confusing aspect of the Hilbert space manual described here is the multiple roles played by the same mathematical object. Indeed, a projection onto a one dimensional subspace plays four distinct roles: (1) it is an outcome, (2) it is a proposition, (3) it is a state or weight, and (4) it is an observable. The danger here is that, for example, a superposition of states is mathematically equivalent to a superposition of outcomes, propositions, etc.

3. A SOCIAL SCIENCE EXAMPLE: MEMORY EXPERIMENTS

3.1. Memory Experiments as Social Science

The study of human memory is a typical social science, and the study of basic memory processes via experiments on learning word lists would seem an uncomplicated example. Yet the complexities are quite marked on both the theoretical and applied sides.

Common sense suggests that events are remembered by storing a memory trace in the brain and some memories are stronger than others. The “strength of the memory trace” should predict memory performance. Research suggests that this simple and intuitively appealing “theory” is inadequate. In particular, data on false memories—remembering things that did not happen, say, remembering that you sent a letter when you only “wrote it in your head” or remembering false lyrics to a song—suggest the need for a more complex theory of memory.

One such theory is Fuzzy Trace Theory, which asserts that people remember things about the verbatim details of a situation stored in a verbatim trace, but that they also abstract the overall gist or meaning of the situation and separately, and independently, store this gist or fuzzy trace; subsequent memory performance is based on both traces (Brainerd *et al.*, 1999; Brainerd and Reyna, 2001; summarized in Brainerd and Reyna, 2002). The verbatim trace has been found to fade more quickly than the gist trace, so after a time, the exact words spoken might be forgotten, but the meaning of what was said remains: The initial statement “I was a math major in college” might be remembered as “I studied math in college” but not “I learned hair styling in beauty school.” In the type of memory experiment discussed below, subjects study a list of words from specific categories.

For example, they might study HORSE, COW, DOG, APPLE, BANANA, PEAR. The verbatim trace would be exactly this list of six words. The abstracted gist might be “animals and fruits.” If a subject remembers CAT or ORANGE, these are gist-consistent, verbatim-false memories.

As in physics, there are controversies, subtleties and complexities that this portrayal ignores, but it suffices to exemplify a social science research paradigm. It is not important whether Fuzzy Trace Theory is correct or even plausible: It represents a typical social science theory, and its ultimate fate depends on empirical support; assessing whether the data support it requires an adequate statistical description such as the one provided below.

3.2. Recall, Recognition and the Fuzzy Trace Covert Judgments

Memory tasks can be crudely divided into two types: Those that ask you to recognize as a valid or invalid memory something that is proposed to you (Think: Multiple-choice test), and those that ask you to recall specific information (Think: Fill-in-the-blank test). Both types of tasks can be and have been used to study memory, and a viable memory theory must explain both. In what follows I focus on recognition, because it is conceptually and structurally simpler. Continuing the above word list example, a recognition test of memory might ask subjects to respond Yes (i.e., “It was on the list.”) or No (i.e., “It was not on the list.”) to each of the following probe words: HORSE, ORANGE, FLUTE, CAT, HAMMER, PEAR. In this test list HORSE and PEAR are targets, that is, words that were, in fact, on the list. The remaining four words are distractors: CAT and ORANGE are related distractors, while FLUTE and HAMMER are unrelated distractors. (Note that actual study and test lists are usually much longer than this—scores to hundreds of such words from perhaps 6 to 20 categories.)

Fuzzy Trace Theory holds that, when a word is presented as a probe on a recognition test, covert judgments are made about the probe based on stored verbatim and gist traces, and these covert judgments determine overt behavior, say, identifying the probe as a target, a related distractor or an unrelated distractor. An identity judgment is made if a sufficiently strong verbatim trace of a studied target matches the probe. A similarity judgment is made if a sufficiently strong gist trace of studied targets matches the probe. A nonidentity judgment is made when, based on verbatim information, a probe can be ruled out as a target: If the probe ORANGE called into memory all three of the fruits on the list, then ORANGE could be eliminated as a target. The likelihood of these covert judgments differs for different probe types. If both verbatim and gist traces are strong, identity and similarity judgments would likely be made for targets, and similarity and nonidentity judgments would likely be made for related distractors; for unrelated distractors none of these judgments is likely.

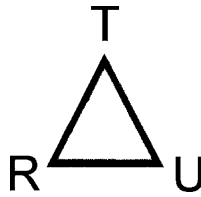


Fig. 1. The TRU triangle.

3.3. The Theoretical Memory Manual

“Systems are prepared” by having subjects listen to the words on the study list. Characteristics of such systems can be examined using various operations. Of interest for the moment are operations asking subjects to identify probes as targets (T), related distractors (R) or unrelated distractors (U). Each operation has three possible outcomes and the frequencies of these under various conditions are studied. The prototypic operation is thus represented by the TRU triangle shown in Fig. 1 at the right. There is a different such operation for each probe type. Hence the manual for the experiments is $\{\{T_T, R_T, U_T\}, \{T_R, R_R, U_R\}, \{T_U, R_U, U_U\}\}$ where R_T is understood to be the outcome of the subject’s saying the probe was a related distractor when in fact it was a target, etc.

Already we are faced with a complication not faced by physics. If a physical system is prepared and electrons are bounced off the system to try to determine its properties, each of the electrons is identical. (Charley Randall: “Electrons don’t have first names.”) In the case of memory experiments, however, various related distractors are not identical. Indeed, asking about ORANGE vs. PERSIMMON will likely produce very different responses, even though both are fruits. Furthermore, not all targets are alike either: Although we speak of the “probability of an identity judgment for a target probe,” this probability differs from target to target, depending, for example, on where the target occurred in the study list and the memorability of the target itself.

To complicate matters further, the preparations are “expensive” in the sense that some human must sit and study the words to be learned, then respond to the test. The supply of people willing to do such things is often limited to the size of a Psychology 101 class for a given semester. In light of this, each “system” (i.e., subject) is tested with multiple probes, therefore, the actual experiment is a compound experiment. Past research has shown that each probe disturbs the system, so that the system is in a slightly different state after each probe. Although these perturbations are occasionally themselves of interest, more often they are nuisances that are statistically averaged out, say, by randomizing the order of the probes for different subjects, then collapsing responses within probe type. After collapsing, the manual for the experiment consists once again of three trichotomies

$\{\{T_T, R_T, U_T\}, \{T_R, R_R, U_R\}, \{T_U, R_U, U_U\}\}$, where R_T is now understood to be an idealized outcome whose frequency is the proportion of times the subject said a target was a related distractor.

This idealized memory manual is used in what follows, but it is important to consider its arbitrary nature. If related distractors came in two varieties, highly related (CAT, ORANGE) and somewhat related (SKUNK, CORN), there would be four probe types, hence four TRU triangles, in the manual (or four $TR_{hr}R_{sr}U$ rectangles, if subjects are to make this discrimination). Since every possible probe is different from every other, there are as many possible probe types as there are words (and non-words are used in some variants), hence the possible number of operations that could be carried out on a preparation is large and unknown (and for words, varies with time and depends on the language spoken by the participant). Moreover, the number of outcomes could be three or four or more, in contrast to the spin-one manual where all (maximal) operations have three outcomes (and to continua in physics where any subdivision is possible, whereas for memory research the ability to discriminate breaks down if levels of relatedness are too close). And worse yet, experiments are always carried out on “mixed beams,” since not all “preparations” are in the same state: Some participants “zoned out” as certain words were read; some may not know some words (we try to avoid this generally); some may not pay attention to the response instructions; some may have emotional reactions to some fruits—i.e., like or dislike them—and hence respond in an idiosyncratic way; or some words may be more accessible because they were used more recently. Changing one word on the study list, or the order of the words, means the preparation is different. We can do the same “counter balancing” act as for test lists, but again there are very large number of preparations and the relationship between two preparations is likely to be ordinal (B is between A and C) rather than metric (A differs from B by x units on some scale). Furthermore, different subjects abstract—and hence remember—different gists (say, “animals” vs. “domestic animals” or “fruits” vs. “edible plants”).

3.4. The Coarsened Memory Manual

For technical and historical reasons, recognition experiments are typically carried out as Yes-No experiments rather than the three-way classification suggested in the last section. Thus, the three-way classification $\{T, R, U\}$ is coarsened into the three two-way classifications $\{T, T'\}$, $\{R, R'\}$ and $\{U, U'\}$, where T' means “not a target,” etc. There are three operations corresponding to each of these decision dichotomies, one for each probe type, so the overall manual consists of nine two-outcome experiments: $\{\{T_T, T'_T\}, \{R_T, R'_T\}, \{U_T, U'_T\}, \{T_R, T'_R\}, \{R_R, R'_R\}, \{U_R, U'_R\}, \{T_U, T'_U\}, \{R_U, R'_U\}, \{U_U, U'_U\}\}$. As was true with the TRU trichotomies above, these operations are to be thought of as idealized operations formed by compounding multiple probes in counterbalanced order and collapsing

by probe type. (It also turns out that subjects often get confused if they are asked to change from one discrimination decision to another, so in practice one group of subjects makes all of the T vs. T' classifications, another group makes all the R vs. R' decisions, and a third group makes all the U vs. U' decisions.)

A weight function on this manual is characterized by its value on the first outcome of each dichotomy (column 1 of Table I). There are nine degrees of freedom— $\Omega \equiv [0, 1]^9$ —since there is no connection in the structure of the manual between any two operations, hence no structural constraints on the weight functions. (This is, of course, identical to the situation for the \mathbb{C}^2 Hilbert space manual, and the reason that Gleason's Theorem does not work there.) Similarly, the logic of this semi-classical manual is nine copies of the Boolean algebra 2 pasted together horizontally.

According to the simplified version of Fuzzy Trace Theory presented here, the conditional empirical frequencies in column 1 of Table I are determined by the probabilities of four covert judgments:

- ι_t = probability that an identity judgment is made for a target probe
- σ_t = probability that a similarity judgment is made for a target probe given no identity judgment
- ν_r = probability that a nonidentity judgment is made for a related distractor probe
- σ_r = probability that a similarity judgment is made for a related distractor probe given no nonidentity judgment

Column 2 of Table I gives the equations relating covert probabilities to overt frequencies. The theoretically meaningful states (cf "physical states") are ones that are generated by this theoretical model. Via these equations, the nine-dimensional weight space for the manual is reduced to a four-dimensional theoretical subspace. The theory is evaluated by collecting empirical weight function data (via compounding and collapsing as above), then estimating the best theoretical parameters to fit this data (iterative maximum likelihood estimation), and finally comparing the empirically observed frequencies with the frequencies predicted by the equations. The predictions are generally within a couple percentage points of the observed values (e.g., Brainerd *et al.*, 2001, Table I).

If memory is perfect, all four theoretical parameters are one, i.e., $(\iota_t, \sigma_t, \nu_r, \sigma_r) = (1, 1, 1, 1)$, and the weight function ω_p is given by the third column in Table I. If there is no memory for the studied words, all four theoretical parameters are zero, i.e., $(\iota_t, \sigma_t, \nu_r, \sigma_r) = (0, 0, 0, 0)$, and the weight function ω_0 is given by the fourth column in Table I. Since memory for verbatim details fades more quickly than memory for the overall gist, after a time delay starting from perfect memory, ι_t and ν_r will be small, while σ_t and σ_r will remain relatively large. The interesting phenomenon occurs as one approaches the state of zero verbatim memory and perfect gist memory as happens when the gist is strongly

Table 1. Probabilities Assigned to Different Types of Probes for Each of the Three Dichotomous Discriminations. $\omega_x(Y_Z)$ is the Probability That a Z Probe is Called a Y in the YY' Discrimination Under Conditions x

Theoretical probability	Probability in terms of the theoretical parameters	Perfect gist and perfect verbatim memory (ω_p)	No gist or verbatim memory (ω_0)	Perfect gist memory and no verbatim memory (ω_g)
$\omega(T_T) = P("T" T)$	$t_t + \frac{(1-t_t)\sigma_t}{(1-t_t)\sigma_t}$	1	0	1
$\omega(R_T) = P("R" T)$	$\frac{(1-t_t)\sigma_t}{(1-t_t)(1-\sigma_t)}$	0	0	1
$\omega(U_T) = P("U" T)$	$\frac{1 + (1-t_t)\sigma_t}{1 + (1-t_t)\sigma_t}$	0	1	0
Sum		1	1	2
$\omega(T_R) = P("T" R)$	$\frac{(1-v_r)\sigma_r}{v_r + (1-v_r)\sigma_r}$	0	0	1
$\omega(R_R) = P("R" R)$	$\frac{v_r + (1-v_r)\sigma_r}{(1-v_r)(1-\sigma_r)}$	1	0	1
$\omega(U_R) = P("U" R)$	$\frac{1 + (1-v_r)\sigma_r}{1 + (1-v_r)\sigma_r}$	0	1	0
Sum		1	1	2
$\omega(T_U) = P("T" U)$	0	0	0	0
$\omega(R_U) = P("R" U)$	0	0	0	0
$\omega(U_U) = P("U" U)$	1	1	1	1

Note. The Sum rows represents the total conditional probability for the operation $\{T, R, U\}$ when these outcomes are identified with the like-labeled outcomes in the dichotomies $\{T, T'\}$, $\{R, R'\}$, and $\{U, U'\}$. The formulae in column 2 predict actual behavior; see text for definitions of the parameters. The underlined terms represent the oddities of behavior that cause the incompatibility of the operations; see text for details. In this setup the four theoretical parameters are estimated from the nine empirical probabilities. In real experiments, subjects sometimes identify unrelated distractors as targets or related distractors, for no obvious reason, so three additional "bias" parameters are needed, one for each discrimination, resulting in seven parameters to be estimated from nine data points (cf. Brainerd *et al.*, 1999).

reinforced (by, say, presenting many exemplars of the given category) and there is a long delay between study and test, allowing verbatim memory to fade. In this case $(\iota_t, \sigma_t, \nu_r, \sigma_r) = (0, 1, 0, 1)$, so, according to the above equations—and in agreement with empirical data—the weight function is given by ω_g in the last column in Table I.

Behavior based on verbatim information is easy to predict and understand; it is the way gist information is used that produces the incompatibilities in the operations. Without verbatim information, it is impossible to discriminate between related distractors and targets, so how do people respond when a similarity judgment is made, but neither an identity judgment nor a nonidentity judgment is made? It depends on the discrimination they are making. If asked whether a probe is a target or not—the T vs. T' discrimination—people accept the probe as a target. If asked whether a probe is a related distractor or not—the R vs. R' discrimination—people accept the probe as a related distractor. The term $(1 - \iota_t)\sigma_t$ therefore appears in both $P("T"|T)$ and $P("R"|T)$. Similarly, the term $(1 - \nu_r)\sigma_r$ appears in both $P("T"|R)$ and $P("R"|R)$. If the outcomes in the dichotomous discriminations are identified with the corresponding outcomes in the three-way TRU discrimination, then the conditional probabilities for each probe type should add to one—but they do not, as indicated in the Sum rows in Table I. The sum exceeds one for targets by the doubled term $(1 - \iota_t)\sigma_t$ and for related distractors by the doubled term $(1 - \nu_r)\sigma_r$. If either verbatim information is perfect (so $\nu_t = \nu_r = 1$) or gist information is missing (so $\sigma_t = \sigma_r = 0$), then both terms are 0 and the sum over TRU is 1, as expected. In most real situations, the sum exceeds 1 and equals 2 in the extreme situation represented by ω_g .

Why might people would behave in this “illogical” way? It is easy to see how evolution might have favored this behavior: Since much real-life memory occurs after a delay when verbatim memory is weak yet gist is strong, we might live our lives as “something like that happened last week.” Although most people will back off to a position such as this when pressed, normally we easily accept verbatim assertions as correct if the overall Gestalt matches the remembered gist sufficiently well. Think back to earlier in this paper: Did I say “I majored in math at college”?

The upshot of the behavior for the manual is (1) that the TRU triangle cannot be added to the manual of dichotomies (if the outcomes are identified) without losing empirically observed weight functions, and (2) that coarsening the TRU trichotomy to $\{T, T'\}$ is not equivalent to coarsening TRU to $\{T, \{R, U\}\}$: When the outcomes R and U are “packed together” to form T' , they behave differently than the event $\{R, U\}$, wherein the subject makes the TRU discrimination, but then R and U are lumped together for recording. A similar phenomenon was noted by Tversky and Koehler (1994, abstract) in people’s judgments of frequency: “. . . judged probability increases by unpacking the focal hypothesis and decreases by unpacking the alternative hypothesis.” The present

situation extends this finding beyond judgments of frequency to acceptance of probes. Although packing R and U into T' has none of the algebraic flavor of adding projections in a Hilbert space, it is in some sense a “coherent-like” coarsening in that it behaves differently if we cannot know which of the packed outcomes occurred, just as an interference pattern is observed in the two-slit experiment if we cannot know which hole the electron went through (ontologic uncertainty), but not if we could know but did not bother to look (epistemic uncertainty).

4. COMPARING QUANTUM MECHANICS AND SOCIAL SCIENCE

4.1. The Manual

In the empirical sciences, a well founded experimental program ultimately is concerned with some cohesive collection of physical operations—usually complete or exhaustive in some sense. Randall and Foulis (1978, p. 170)

The manual of operations for spin-one particles is completely known in the sense that the in-principle measurements are represented by the self-adjoint operators on \mathbb{C}^3 , and the in-principle manual of elementary operations consists of all maximal sets of mutually orthogonal \mathbb{C}^3 projections. Indeed, every experiment already has a name, to wit, the set of matrices representing the projections in the operation. Real life measurements correspond to one of these in-principle measurements in more or less known ways, and any in-principle measurement can be realized by a suitable generalized Stern-Gerlach (GSG) apparatus (Swift and Wright, 1980). Although not all possible real-life measurements are known, any such measurement is presumed to be equivalent to one of these GSG measures via the correspondence with Hermitian operators. Similarly, the relationship between any two in-principle measurements, and hence between any two real-life measurements, is known, and the experiments are intricately intertwined. Any two one-dimensional outcomes occur in overlapping operations: The outcome orthogonal to both will be the overlap. (This overlap is what constrains possible weight functions and makes Gleason’s Theorem work.) The manual of in-principle operations and—up to GSG equivalence, the manual of real-life operations—is exhaustive and coherent, as Randall and Foulis required. The manual is parameterized by the field of the GSG.

Experiments in the memory manual can also be changed in “parametric” ways paralleling rotations in space of the Stern-Gerlach apparatus: (1) The preparation of systems can be changed by using different numbers of exemplars of each category; (2) The probes can be changed by varying their prototypicality within the studied category; (3) the setting of the discrimination can be changed by changing the delay between study and test. Moreover, (4) the discriminations to be made can be changed by including more than one level of relatedness of

distractors. There is therefore one memory experiment for each preparation-probe-setting-discrimination combination.

Although such combinations describe all experiments involving studying words from categories and responding to recognition probes, unlike the spin-one situation, what variations are possible is not fully knowable. We know some variations, but some differences once considered irrelevant are later found to be important, introducing new variations. If the experiments to be generalized to are “memory for words from categories,” then the specific categories used should not matter. Yet research has shown that concrete categories (e.g., fruits) and abstract categories (e.g., personality traits) behave differently, so to handle all memory experiments, the manual’s parameterization needs an added dimension for abstractness of the categories. Even beyond adding new dimensions there are problems with identifying all possible preparations: What constitutes a word? Is a word a particular participant has never heard, a word for purposes of testing that participant’s memory? How long can a word list be? If it takes longer than a human lifetime to listen to it, it is not a real experiment. Furthermore, German word lists correspond to a different manual than English word lists. In sum, the memory manual is unknown and additions and refinements to the manual are continually being made. In particular, the manual considered by researchers at a given moment in time is not exhaustive.

There are theoretical relationships between measurements and outcomes in memory research just as there are in physics, but the experiments are not as intricately intertwined, and relationships are generally ordinal rather than metric, in contrast to the spin-one situations where rotating the GSG apparatus by a certain amount has a precisely predicted effect. For example, experimentally manipulating the number of exemplars per category—say 5 vs. 3—will change the memory traces, strengthening the memory trace for the category as a whole (hence increasing σ_t and σ_r) but having little effect on ι_t for individual, once-presented targets. The effect on σ_t and σ_r using 4 exemplars per category would be between these two, but there is no prediction of exactly how much. Hypotheses then typically have the form of “such-and-such manipulation causes an increase in a certain variable.”

The relationship between real and idealized experiments in memory research is also more complex than in physics. While no physical measurement is perfect, the naive expectation is that given a theoretical measurement, with sufficient care, a real-life measurement as close as desired to the theoretical measurement can be devised (but compare Foulis and Gudder, 2001). In memory research, however, we wish to measure memory for categorized lists in general, whereas all experiments use specific categories that have properties not of theoretical interest. Any real word-memory experiment is presumed to be a poor representative of the theoretical word-memory experiment of interest. In memory experimentation the implicit theoretical manual is the set of experiments the present experiments

are presumed/claimed/intended to generalize to, but the real manual uses a finite number of specific words.

We address this problem by using multiple specific categories and averaging to eliminate the extraneous and retain what results from common factors. A study of memory for fruits or metals would be unconvincing as revealing general memory phenomena, because the results might be influenced by something specific about fruits or metals. It is only when we demonstrate a phenomenon with multiple categories, to show it is independent of the categories chosen, that we have convincing evidence about “memory for categorized lists.” The reasoning here is similar to that of having multiple items on an IQ test: Answering a particular item correctly is influenced by IQ as well as other experiences and abilities; only when we average many of these items together do the other things “average out” leaving a more pure assessment of the underlying IQ contribution.

4.2. Theory and States

In Operational Statistics, content-based theory is introduced in a couple different ways. For both quantum physics and social science, formal and informal theory go into specifying the manual, particularly outcomes that occur in the overlap between operations. An important example is whether T from TRU should be identified with T in TT' . This dose of theory Theory introduced by outcome identifications suffices for spin-one experiments (but not spin-on-half), because the structure of the manual determines exactly the weight functions that correspond to physical states.

Since in memory research there is no *the* manual, its structure cannot determine the states/weights (nor the logic), so theory to specify the meaningful states must be added in some other way. Fuzzy Trace Theory per above, specifies that the meaningful states arise from specific values of t_t , σ_t , v_r , and σ_r via the equations in column 2 of Table I.

States in social science are always partial states: Given the complexity of a human being, we never suspect that we have captured the full state, just as we would not believe we have captured the full state of a feather in a room. In the rarified realm of in-principle spin-one measurements, however, the state of a specific particle (with phase information) is a pure state, and is assumed to be the full and complete description of the particle. For memory experiments, a full description is not even desirable: Since we want to study memory for categorized lists (say), we do not want a state that specifies probabilities for each word, but rather for targets vs. related distractors (say). In preparing spin-one particles, each particle is in some pure state, with an ensemble of particles forming a “mixed beam” of known distribution. In memory research, not only are all beams mixed (different subjects are in different states), but each individual subject is in a non-pure, partial state; we only have information about the state with respect to certain

specific experiences, allowing multitudes of infinities of actual states, the specifics of which are unknowable and not of interest.

4.3. Coherent Coarsenings (Packing) and Outcome Interference

According to Feynman, the basic features of quantum mechanics are ... a probabilistic theory ... outcomes that interfere with each other ... cannot be distinguished without disturbing the system. Gudder (1988, p. xi)

Memory experiments have the basic feature of quantum mechanics as expressed in the above quote. Memory theories are probabilistic theories. In the memory example above, the outcomes R and U interfere with each other when they are packed together as T' in the sense that the probability of T' is not generally equal to the probability of the event $\{R, U\}$, which represents distinguishing between R and U experimentally.

The above quote continues with Feynman's belief that "The main feature of quantum mechanics is the way that probabilities are computed" (Gudder, 1988, p. xi), namely, taking the squared modulus of complex amplitudes: If the coarsening is coherent (outcomes interfere), the amplitudes are added and then the squared modulus is taken; if the coarsening is incoherent (no interference), the squared modulus is taken before adding. Note that in computing these quantum probabilities, there are exactly two versions: Add first or add second.

If one takes the basic features of "probabilistic theory with interfering outcomes" to be the defining characteristic of nonclassical theories (or general quantum structures), and "computing probabilities via amplitudes" as defining a specifically quantum theory, then memory theories are nonclassical but not quantum. The packed coarsening of R and U into T' resembles the "coherent" coarsening in quantum mechanics in that the probability is different from the probability of the event $\{R, U\}$, but probabilities in this situation cannot arise from amplitudes, even if one were to allow amplitudes to come from some structure more general than \mathbb{C} and have a more general mapping into probabilities than squared modulus. To see this one need only note a sense in which the memory example is even less classical than quantum mechanics: In comparing the experiments $\{T, T'\}$ and $\{T, \{R, U\}\}$, not only are the probabilities of T' and $\{R, U\}$ different, but (consequently) the probability of T is different in the two experiments, hence the probability of T cannot arise from an underlying amplitude for T . If the spin-one experiment $\{P1, P2, P3\}$ is coarsened into $\{P1, P2 \oplus P3\}$ and $\{P1, \{P2, P3\}\}$, the probability of $P1$ remains unchanged. Indeed, the probability of $P2 \oplus P3$ is always the same as the probability of $\{P2, P3\}$, because it is only in compound experiments that the difference between coherent and incoherent coarsenings shows itself (Wright, 1978a).

The oddity in the memory experiments arises because of the identification of the outcome T in TRU with the T in TT' , but people seem to experience these T s as equivalent. Yet empirically, packing the outcomes R and U changes the identity of both the packed T' and unpacked T alternatives. So, subjects experience T as the same in TT' vs TRU , but assign it different probabilities! The oddity, therefore, is in the way that human brains process information, rather than (at least in any direct way) a macroscopic manifestation of some true quantum phenomenon (cf. Hagan et al., 2002). This also puts the locus of the oddity in a vastly more complex system (the brain) that is likely to behave in more unruly ways than spin-one particles.

That humans are making judgments also has ramifications for compound experiments. It might be of interest, for example, to know what would happen if the TT' experiment was followed by the TRU experiment. If the classification is to be made about the same probe, the question becomes “You said HAMMER was a target, now say whether it is a target, a related distractor or an unrelated distractor.” The subject’s response is likely to be: “I just told you it is a target.” But moreover, once the TRU discrimination is introduced, it is likely to unpack R and U in the subject’s mind, changing subsequent judgments in the TT' discrimination. This is a further sense in which the TT' and TRU experiments are incompatible. And TT' and RR' may be incompatible for similar reasons: They imply the “common refinement” TRU , thereby unpacking T' and R' and changing the original dichotomies. It is exactly this sense in which TRU is not a common refinement of TT' and RR' . It is also interesting to note here that unpacking might not be complete, allowing for intermediates between the fully packed outcome T' and the fully unpacked event $\{R, U\}$, in contrast to the dichotomous nature of coarsenings in quantum mechanics, i.e., whether you take the squared modulus before or after adding the amplitudes.

Given the influences on human thinking, it may not be possible to do any compound operations on the same probe, in stark contrast to physics where a particle coming out of one Stern-Gerlach apparatus can be fed unproblematically into a second. Indeed, the physical theory explaining one such experiment generally allows us to predict the outcomes of compound experiments without having a specific theory of them. This generalization in physics from simple to compound operations is often based on the idea that measurements are of the “first kind” (Pauli, 1958). Are memory measurements of the first kind? The “I just told you . . .” response suggests they might be, but the resulting state is not pure as it often is in physics, and if, as just discussed, compound operations in memory research is problematic, how can we base the definition of “first kind” on empirical data?

4.4. Other Issues

There are several other issues worth considering briefly because they are often raised as prototypic of quantum mechanics or social science. For example:

Do superpositions of states occur in social science? If a superposition is a linear combination of vectors in a complex state space, then this question is meaningless, since there are no complex state spaces for social science experiments. If the question is about some generalization of the concept of superposition such as the one proposed by Foulis *et al.* (1983, p. 823), then certainly these exist in social science; indeed ω_p is a superposition in this sense of ω_0 and ω_g , as is any weight that treats unrelated distractors as such, no matter what it does for targets and related distractors. What is less clear is whether this tells us anything useful.

Another difference between quantum mechanics and social science, related to complexity, is the question of isolated systems. In both quantum and classical mechanics, we perform Gedanken experiments on isolated systems and fancy that we can create a system as nearly isolated as needed to check out the theory. Social science deals with open, non-isolated systems. In the memory experiments, for example, subjects come to the experimental sessions with a lifetime of using the words that they will be tested on, and recent usage outside the lab may parallel studying the word, so the outcome of the experiment is influenced by extra-experimental factors which can never be eliminated. As was the case with having to use specific words to test types of words, we dampen extra-experimental influences by averaging over many subjects, no one of which is an isolated system.

Both physics and social science use mathematical models—e.g., the Hilbert space model or the Fuzzy Trace model—but the understanding of what the models represent is different. In physics, a model is taken to represent reality which actual experiments approximate, and although there are different Hilbert space models for different systems, a given system has a specific mathematical model. A physical state is a full description of the system at a given moment. In social science, models are generally not representative of reality, but merely descriptive: Things work out as if this were true. If a model is empirically supported, any theory must be consistent with this model to be credible. Fuzzy Trace Theory does not specify a specific mathematical model for memory experiments, but rather offers principles for how to construct models for specific situations. The model presented above provides good predictions for the memory of words in categories, but if the situation is changed—e.g., if there were two levels of relatedness of distractors—then Fuzzy Trace Theory would add other parameters.

5. SUMMARY AND AGENDA FOR FUTURE RESEARCH

It is all too common these days for people to add “quantum” to a concept just for the cachet of the word, for example “quantum healing” (healing that occurs in tiny quanta?). That is not my intention here. I am not suggesting that social science is quantum like. Rather, I am suggesting that quantum mechanics has forced us to consider nonclassical logic and statistics, and that social science is even more nonclassical than quantum mechanics. Quantum mechanics and

social science are similar in that they can both be described by the Foulis-Randall Operational Statistics model, they are both inherently probabilistic (for different reasons), they both deal with incompatible experiments and they both exhibit outcome interference. And we have seen that Nature does “take advantage of” the incompatible experiments by frequencies incompatible with a common refinement.

Yet, quantum mechanics and social science are also fundamentally different in that quantum mechanics provides a rich, complete mathematical description of simple, isolated systems resulting in a small set of physical laws about a manageable number of elementary particles and forces, whereas social science deals with sparse, partial descriptions of complex, open systems, resulting in many approximate mini-theories about vast numbers of “objects,” for example, the unknown and unknowable number of words in the memory experiments.

Fuzzy Trace Theory in particular has much in common with quantum theory. Both began with facts that did not make sense given understandings at the time (e.g., interference patterns in the two-slit experiment or acceptance of related distractors as targets in memory research). Both proposed unobservable mathematical features (amplitudes in physics, probabilities of covert judgments in memory) and equations that predict observed phenomena from these unobservable features. In both domains, theories sink or swim based on whether they predict observations across situations. Both predict that states evolve over time, based on the unobserved features, and each in its characteristic way: Quantum mechanics specifies a Hamiltonian describing metrically how the state evolves; Fuzzy Trace Theory specifies ordinally how parameters change with delay between study and test. The similarities between quantum mechanical and social scientific theories make it tempting to take mathematical structures derived to describe quantum structures and apply them, or adapt them, to the social sciences. This paper suggests, however, specific challenges to researchers who desire to build a foundation—as Randall and Foulis (for example, 1978, as quoted above) proposed to do—for all empirical science. I list some key challenges here:

1. We cannot assume that the manual of all relevant experiments is known, and if the manual is not exhaustive, we cannot judge its coherence or other similar properties. Without knowing the manual, we cannot know the logic, so we cannot ask about its properties, for example, “Is the logic an orthomodular poset?”
2. Theoretical considerations have to be brought in exogenously to the structure of the manual itself, for example, via the parameters and equations derived from Fuzzy Trace Theory.
3. Outcome interference is less neat in social science than in quantum mechanics (for example, the graded packing of R and U described above), occurs for different reasons (namely the way human brains work rather

than the nature of the universe directly), and may change frequencies of non-coarsened outcomes.

4. The relation of real experiments to theoretical experiments is more complex in social science, for example, having to test memory for categorized words using specific words from specific categories. To approximate theoretical outcomes we must average over multiple subjects, study lists and probes, none of which is in itself a particularly good example of the entity of interest. (One may average in physics, too, but often over replications of the same experiment).
5. Theoretical relationships and empirical hypotheses are mostly ordinal, not metric. For example, using a longer study list might be predicted to change overall memory performance, but not by a specific amount.
6. Mathematical models in social science are approximations to real data, not truth. States are partial states on open systems.

The dismissive view of psychology as soft science leads to the spurious question: Can psychology become like physics when it grows up? I hope that the above thoughts can allow physicists to move beyond this stereotypic view of social sciences as merely sloppy physics, and can offer those working in quantum structures a vision of the territory that must be traversed before a true description of empirical science is in hand. And that leaves the final question of whether the new description can do real work in social science.

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